

The Dynamics of Poverty and Crime

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Abstract

Poverty and crime are two maladies that plague metropolitan areas. The economic theory of crime (Becker, 1968) demonstrates a direct correlation between poverty and crime. The model seeks to examine the dynamics of the poverty-crime system through stability analysis of a system of ODEs in order to identify cost-effective strategies to combat crime in metropolises.

1 Introduction

There is a direct correlation between poverty and criminality (Kelly, 2000; Block and Heineke, 1975). Becker's economic theory of crime (1968) assumes that people resort to crime only if the costs of committing the crime are lower than the benefits gained. Those living in poverty, therefore, have a much greater chance of committing property crime (Kelly, 2000, Chiu and Madden, 1998) than the general population. Property crime is defined as burglary, larceny, or theft (O'Connor, 2005). In his 1968 paper, Becker used statistical and economic analysis to determine the optimal control of crime. Here, we use a system of ODEs to try and get a more realistic, dynamical solution to that same question.

Property crime is a major problem in metropolises. In the Bronx borough of New York City alone there were 247 reported complaints of property crime in one week (NYPD, 2005). There are over 36,000 cases of property crime reported in one year (NYPD, 2005). The Bronx also has a poverty rate of 37% (Kids Well-Being Indicators Clearinghouse, 2005) and has a population of over 1.3 million (US Census Bureau, 2000).

Each criminal costs society about \$5,700 per year due to lost productivity (Kelly, 2000), and a total of \$24 billion in goods is lost in the US each year to property crime (US Dept. of Justice, 2004). The victims of crime suffer an aggregate burden of \$472 billion per year, including mental and physical suffering (Anderson, 1999).

Crime is clearly an important problem that must be confronted. Ehrlich (1981) suggests that the successfulness of rehabilitation and incapacitation programs do have an effect on the aggregate crime level. However, it costs about \$25,000 to detain a person in a federal prison each year (US Dept. of Justice, 2003). It also costs about \$100,000 to build a new prison cell, and the prison population is growing rapidly (The Special Committee on Drugs and the Law of the Association of the Bar of the City of New York, 1994). We see that from an economic standpoint, detaining every prisoner is actually a greater burden on society than crime itself is. Therefore, the issue becomes one of balance. Ultimately, the goal is to reduce crime to such a level that the total cost of controlling crime and the cost of the crime that remains is less than the total cost of crime under the status quo.

What this model aims to do is find a cost-effective way to lower criminality, thus lowering the cost of crime to society. Previous works have addressed this problem (Becker, 1968, Ehrlich, 1973) using statistical and economic approaches, while we do so by taking a mathematical (dynamical systems) approach. The use of ODEs allows us to examine the dynamics of the poverty-crime system and gives us a changing, rather than static, view of how criminality is affected by varying intervention parameters. We know that the problem of crime is alleviated by either decreasing poverty (Witte, 1980) or by increasing the severity of the ensuing punishment (Block and Heineke, 1975). Our model considers both solutions concomitantly. Logically, crime will decrease if one or the other intervention parameters is increased. However, because we seek a pragmatic solution in a world where resources are limited and cost is always a consideration, we cannot reduce crime by simply relieving all poverty or by incarcerating all criminals. Instead, we seek a cost-effective strategy to combat crime. Our model shows that the optimal solution is actually a combination of the two control parameters and pinpoints where that solution is.

2 Model

Naturally, not all crime can be stopped; that would not be economically desirable. This model seeks to optimize interventions so that crime is rea-

sonably controlled and so that the cost is minimal. The two interventions considered are γ , the rate of converting those in poverty to recovered, and ρ , the rate of incarceration. In the model, the population is divided into five sub-classes: the non-impooverished class N , the poverty class P , the criminal class C , the jailed class J , and recovered class (from jail or from impooverished class) R . The total population is $T = N + P + C + J + R$.

Let σ denote the rate of the flow from the non-impooverished class to the impooverished class. It is assumed that sigma is omnipresent and dependent upon the unemployment rate because of the nature of unemployment and because of the dependency of poverty on unemployment. γ denote the conversion rate from the P class to the R class due to government interventions; ρ the rate at which criminals is captured; δ denote the rate at which individuals get out jails; μ is the death rate, and since T is constant ($dT/dt = 0$), μ is also the birth rate. All rates are per capita.

We assume that there is a certain probability that a person in the P class will resort to crime after coming into contact with a criminal. The term $\beta PC/T$ is the conversion of impooverished individuals to criminals due to contact over a certain period of time. β represents the "transmission" rate. A recovered individual may also become criminal again but at a reduced rate $\phi\beta RC/T$ where $0 \leq \phi \leq 1$ is the reduction fraction that accounts for recidivism. The known rate of recidivism is around 50% (Bureau of Justice Statistics, 2005). The assumption is that those who have already gone to jail flow immediately into the R class, and then, due to contact with criminals, revert back to criminality at some reduced rate $\phi\beta$. The rate is reduced because these people have a greater cost to commit their next crime, according to Becker's theory (1968). All parameters are assumed non-negative. Under these assumptions the interaction between poverty and crime is governed by the following system:

$$\begin{aligned}
 N' &= \mu T - (\sigma + \mu)N \\
 P' &= \sigma N - \beta P \frac{C}{T} - (\gamma + \mu)P \\
 C' &= \beta P \frac{C}{T} + \phi \beta R \frac{C}{T} - (\rho + \mu)C \\
 J' &= \rho C - (\delta + \mu)J \\
 R' &= \gamma P + \delta r J - \phi \beta R \frac{C}{T} - \mu R \\
 T &= N + P + C + J + R.
 \end{aligned}
 \tag{1}$$

The reproductive number is

$$\mathcal{R} = \mathcal{R}_P + \mathcal{R}_R \quad (2)$$

where

$$\mathcal{R}_P = \frac{\beta}{\rho + \mu} \frac{\sigma}{\sigma + \mu} \left(1 - \frac{\gamma}{\gamma + \mu} \right), \quad (3)$$

$$\mathcal{R}_R = \frac{\phi\beta}{\rho + \mu} \frac{\sigma}{\sigma + \mu} \frac{\gamma}{\gamma + \mu}.$$

\mathcal{R}_P and \mathcal{R}_R represent the contributions from the P and R classes, respectively. The factors $\beta/(\rho + \mu)$ and $\phi\beta/(\rho + \mu)$ give the numbers of new criminals from the P and R classes respectively produced by one criminal individual during the entire criminal period before being captured, and the factor $\sigma/(\sigma + \mu)$ gives the probability that a non-impooverished individual survived and entered the impoverished class while $1 - \frac{\gamma}{\gamma + \mu}$ and $\frac{\gamma}{\gamma + \mu}$ represent respectively the probabilities that a P individual remains in the P class or has moved into the R class.

3 Mathematical analysis

In this section we consider possible equilibria and their stability. Since T is constant the expressions are simplified if we look at the fractions. The *CFE* (crime-free equilibrium)

$$E_0 = \left(\frac{N_0}{T}, \frac{P_0}{T}, \frac{C_0}{T}, \frac{J_0}{T}, \frac{R_0}{T} \right) = \left(\frac{\mu}{\sigma + \mu}, \frac{\mu\sigma}{(\sigma + \mu)(\gamma + \mu)}, 0, 0, \frac{\gamma\mu\sigma}{\mu(\sigma + \mu)(\gamma + \mu)} \right)$$

always exist.

Let $E^* = (N^*/T, P^*/T, C^*/T, J^*/T, R^*/T)$ with $C^* > 0$ denote a crime equilibrium. Setting the right-hand side of Eq (1) equal to zero we can express all components of E^* in terms of $x = C^*/T$:

$$\frac{N^*}{T} = \frac{\mu}{\sigma + \mu}, \quad \frac{P^*}{T} = \frac{\mu\sigma}{(\sigma + \mu)(\gamma + \mu + \beta x)}, \quad \frac{J^*}{T} = \frac{\rho x}{\delta + \mu} \quad (4)$$

$$\frac{R^*}{T} = \frac{1}{\mu + \phi\beta x} \left(\frac{\gamma\mu\sigma}{(\sigma + \mu)(\gamma + \mu + \beta x)} + \frac{\delta\rho x}{\delta + \mu} \right).$$

Since the components of E^* add up to one we get the equation for x :

$$\begin{aligned} \frac{\mu}{\sigma + \mu} + \frac{\mu\sigma}{(\sigma + \mu)(\gamma + \mu + \beta x)} + x + \frac{\rho x}{\delta + \mu} \\ + \frac{1}{\mu + \phi\beta x} \left(\frac{\gamma\mu\sigma}{(\sigma + \mu)(\gamma + \mu + \beta x)} + \frac{\delta\rho x}{\delta + \mu} \right) = 1 \end{aligned} \quad (5)$$

or equivalently

$$a_3 x^3 + a_2 x^2 + a_1 x + a_0 = 1, \quad (6)$$

where

$$\begin{aligned} a_0 &= \frac{\mu^2(\gamma + \mu)}{\sigma + \mu} + \frac{\mu^2\sigma}{\sigma + \mu} + \frac{\mu\sigma\gamma}{\sigma + \mu} - \mu(\gamma + \mu), \\ a_1 &= \frac{\mu}{\sigma + \mu} [\phi\beta(\gamma + \mu) + \beta\mu] + \left(1 + \frac{\rho}{\delta + \mu}\right) \mu(\gamma + \mu) + \frac{\sigma\mu}{\sigma + \mu} \phi\beta \\ &\quad + \frac{\delta\rho}{\delta + \mu} (\gamma + \mu) - [(\gamma + \mu)\phi\beta + \mu\beta], \\ a_2 &= \frac{\mu}{\sigma + \mu} \phi\beta^2 + \left(1 + \frac{\rho}{\delta + \mu}\right) [\phi\beta(\gamma + \mu) + \beta\mu] + \frac{\beta\delta\rho}{\delta + \mu} - \phi\beta^2, \\ a_3 &= \left(1 + \frac{\rho}{\delta + \mu}\right) \phi\beta^2. \end{aligned} \quad (7)$$

It is easy to show that $a_0 = 0$. Hence Eq. (6) can be reduced to a degree 2 polynomial (ignoring the $x = 0$ solution)

$$Ax^2 + Bx + C = 0, \quad (8)$$

where $A = a_3$, $B = a_2$, $C = a_1$. Using the expressions in (3) we can rewrite some of these expressions which provide an easier presentation. For example,

$$\begin{aligned} C &= (\rho + \mu)(\gamma + \mu) - \frac{\beta\sigma\mu}{\sigma + \mu} - \frac{\phi\beta\sigma\gamma}{\sigma + \mu} \\ &= (\rho + \mu)(\gamma + \mu)(1 - \mathcal{R}), \end{aligned} \quad (9)$$

$$B = \phi\beta(\gamma + \mu) \left(1 + \frac{\rho}{\delta + \mu}\right) + \beta(\rho + \mu) \left(1 - \mathcal{R}_R - \frac{\mu}{\gamma} \mathcal{R}_R\right).$$

For (9) we have used the fact that

$$\frac{\phi\beta\sigma}{\sigma + \mu} = \frac{\gamma + \mu}{\gamma}(\rho + \mu)\mathcal{R}_R.$$

Let x_{\pm} denote the solutions of Eq. (8), i.e.,

$$x_{\pm} = \frac{1}{2A}(-B \pm \sqrt{B^2 - 4AC}).$$

Notice that $A = a_3 > 0$ for all parameter values. The sign of C depends on the magnitude of \mathcal{R} . We consider two cases.

Case 1: $\mathcal{R} > 1$. In this case, $C < 0$ and $AC < 0$. Hence, x_- is always negative and x_+ is always positive. It follows that Eq. (8) has a unique positive solution and there is a unique positive equilibrium E^* .

Case 2: $\mathcal{R} < 1$. In this case, $C > 0$ and $AC > 0$. Hence, Eq. (8) has either two positive roots if $B < 0$ or no positive roots if $B > 0$. Since $\mathcal{R} < 1$, we know that $\mathcal{R}_R < \mathcal{R} < 1$. If we assume that $\gamma \gg \mu$, then it is easy to see that $B > 0$. Hence, there is no positive equilibria. Therefore the possibility of a backward bifurcation can be ruled out if $\gamma/\mu \ll 1$.

Result 1. System (1) has a unique crime equilibrium E^* if $\mathcal{R} > 1$ and it has only the CFE if $\mathcal{R} < 1$.

We proceed to show the stability of the equilibria. At the CFE, E_0 , the Jacobian matrix is

$$J = \begin{bmatrix} -(\sigma + \mu) & 0 & 0 & 0 & 0 \\ \sigma & -(\gamma + \mu) & -\beta\frac{R_0}{T} & 0 & 0 \\ 0 & 0 & \beta\frac{R_0}{T} + \phi\beta\frac{R_0}{T} - (\rho + \mu) & 0 & 0 \\ 0 & 0 & \rho & -(\delta + \mu) & 0 \\ 0 & \gamma & -\phi\beta\frac{R_0}{T} & 0 & -\mu \end{bmatrix}. \quad (10)$$

We can show that all eigenvalues of J are negative if $\mathcal{R} < 1$ and that J has one positive eigenvalue if $\mathcal{R} > 1$.

The characteristic equation for the crime equilibrium E^* is a degree 5 polynomial which is too complicated to analyze. Therefore, we explore the stability of E^* numerically. Fig. 1 shows that all solutions converge to E^* as $t \rightarrow \infty$ indicating a global stability of E^* .

Result 2. The crime-free equilibrium is locally asymptotically stable if $\mathcal{R} < 1$ and it is unstable if $\mathcal{R} > 1$. The crime equilibrium E^* is stable when it exists.

The practical implications of these results are given in the next section.

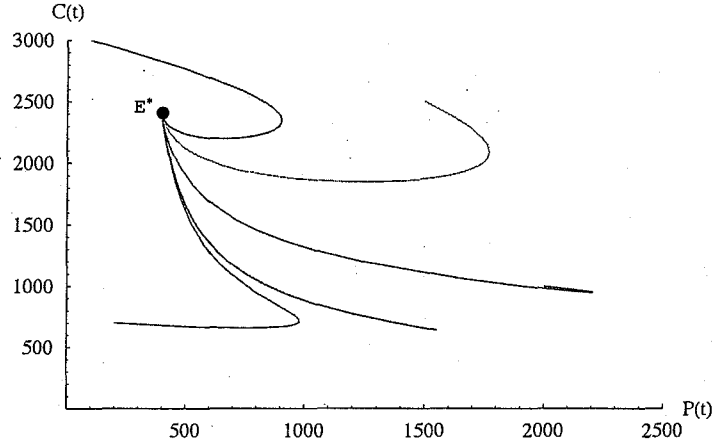


Figure 1: Phase portrait projected to the (P, C) plane for $\mathcal{R} < 1$. It shows that solutions converge to E^* as $t \rightarrow \infty$.

4 Cost-effective crime control strategies

We focus on the scenario when $\mathcal{R} > 1$ in which case the crime equilibrium E^* exists and is stable. At E^* the population size of the P and C classes are

$$P^* = \frac{\mu\sigma T}{(\sigma + \mu)(\gamma + \mu + \beta x)}, \quad C^* = xT,$$

where x is the positive solution of Eq. (8). Clearly, $P^* = P^*(\gamma, \rho)$ and $C^* = C^*(\gamma, \rho)$ are functions of the intervention parameters γ and ρ . Figure 2 shows the dependence of the crime level C^* on γ and ρ . Different combinations of γ and ρ can be used to achieve a given level of crime. However, the costs associated with these control measures might be very different; some may be higher than the background cost while others might be lower.

We compute two costs associated with the crime activity: one is the total cost, denoted by B , in the absence of any additional interventions, i.e., $\gamma = 0$, $\rho = \rho_0$ (a background criminal capture rate) while all other parameters are fixed. The other is the total cost, denoted by \tilde{B} , for a given set of $\gamma > 0$ and $\rho > \rho_0$. There are many different ways of defining the costs. Here, we present only one possible functional form to illustrate how the model results may help gain insights into the problem. Let

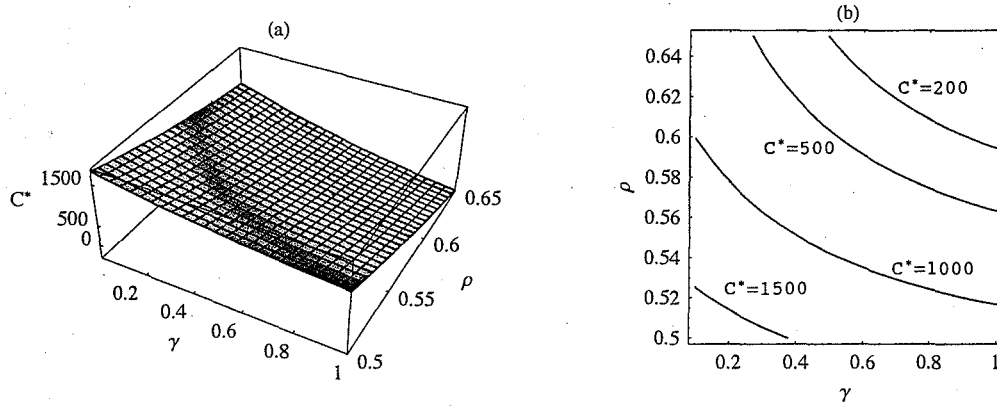


Figure 2: Plot of the crime level C^* as a function of intervention parameters γ and ρ .

$$B = B_1 C^*(0, \rho_0) + B_2 \rho_0 C^*(0, 0),$$

$$\tilde{B} = B_1 C^*(\gamma, \rho) + B_2 \rho C^*(\gamma, \rho) + B_3 \alpha (\gamma P^*(\gamma, \rho))^2.$$

B_1 represents the cost to society associated with each criminal; B_2 is the cost for each captured criminal; and B_3 is the cost coefficient for converting a P individual to an R individual. The first two terms are linear while the last term is a quadratic function of the total number converted (assuming that there are limited resources that can be allotted for this and that the cost is increased at a much faster rate if more people are to be helped). α is a scaling parameter which controls slope of the increase in the cost. Figure 3 demonstrates the relationship between the two costs for a given set of parameters. Fig. 3(a) plots the cost \tilde{B} and Fig. 3(b) shows the regions in which the cost \tilde{B} is greater than the cost B or less than B .

Obviously, a crime control strategy would be to choose $\gamma > 0$ and $\rho > \rho_0$ for which the cost on the red surface is below the cost on the green. Obviously the corresponding crime level $C^*(\gamma, \rho)$ will always be lower than the background crime level $C^*(0, \rho_0)$. For a given prescribed crime level (a fixed contour curve in Fig. 2(b)) the best strategy would be the one that gives the smallest \tilde{B} value along the contour curve.

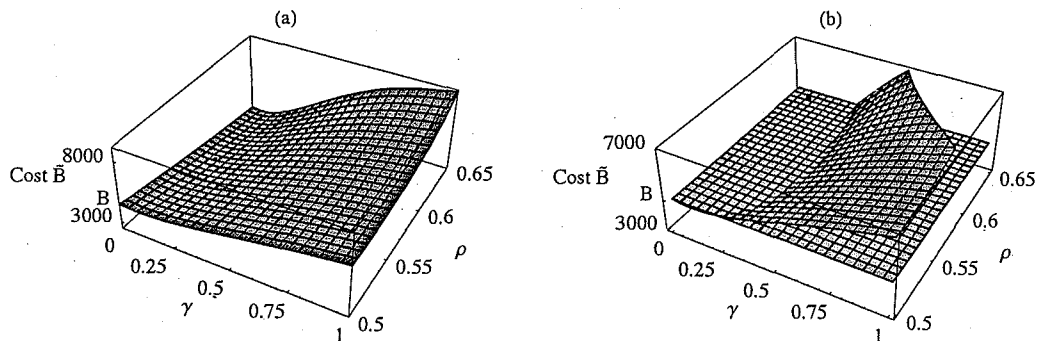


Figure 3: Cost as a function of intervention parameters γ and ρ . The plain is the constant cost $B = 1843$ for $\gamma = 0$ and $\rho = \rho_0 = 0.5$.

5 Discussion

In this paper we developed a mathematical model to study the dynamics of poverty and crime. By studying the property of equilibria and their stability, we derived threshold conditions which can be used to determine the prevalence of criminal activity. That is, the dynamics of the model depend on the reproductive number \mathcal{R} . When $\mathcal{R} < 1$, the crime level will always fall to zero. Therefore, we are only concerned with the case $\mathcal{R} > 1$. When the crime activity is persistent, we explored the possibility of crime control via government interventions without increasing the total cost associated with criminal activity in the absence of intervention programs. We demonstrated that under certain conditions crime control strategies can be identified (see Figure 3) if all the relevant cost functions are known.

The implications of the model are what we expected. Naturally, eliminating all crime is not feasible. The model together with the cost function show that, for a given crime level, there will be optimum values of the parameters γ and ρ , such that the cost of controlling crime is at a minimum. Note that we have only suggested one cost function. Others exist, depending on what controls are to be implied by gamma, among other considerations.

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